# REASONING UNDER UNCERTAINTY:
# FUZZY LOGIC

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# What is Fuzzy Logic?

- What is FUZZINESS?
- According to OXFORD DICTIONARY, FUZZY means

  Blurred, Fluffy, Frayed or Indistinct
- Fuzziness is deterministic uncertainty
- Fuzziness is concerned with *the degree to which events occur* rather than *the likelihood of their occurrence* (probability)

# What is Fuzzy Logic?

- For example:

  The degree to which a person is young is a *fuzzy event* rather than a *random event.*

- Suppose you have been in a desert for a week without a drink and you came upon a bottle A and B, marked with the following information:

  P(A belongs to a set of drinkable liquid) = 0.9

  $\mu$**B** in fuzzy set of drinkable liquid = 0.9

  Which one would you choose?

# What is Fuzzy Logic?

- Some unrealistic and realistic quotes:

  Q: How was the weather like yesterday in San Fransisco?

  A1: Oh! The temperature was -5.5 degrees centigrade

  A2: Oh! It was *really cold.*

  A1: You should start braking at 30% pedal level when you are 10 m from the junction.

  A2: You should start braking slowly when you are *near the junction.*

# What is Fuzzy Logic?

- Expert rely on common sense to solve problem.
- This type of knowledge exposed when expert describe problem with **vague terms**.
- Example of **vague terms**:
  - When it is **really/quite hot ...**
  - If a person is **very tall** he is suitable for ...
  - Only **very small** person can enter into that hole
  - I am **quite young**
  - Mr. Azizi drive his car **moderately fast**
- How can we represent and reason with vague terms in a computer?
- **Use FUZZY LOGIC!!**

# Brief History of Fuzzy Logic

1965 Seminal paper by Prof. Lotfi Zadeh on fuzzy sets

1966 Fuzzy Logic (P. Marinos Bell Labs)

1972 Fuzzy measures (M. Sugeno, TIT)

1974 Fuzzy Logic Control (E.H. Mamdani, London, Q. Mary)

1980 Control of Cement Kiln (F.L. Smidt, Denmark)

1987 Automatic Train Operation for Sendai Subway, Japan (Hitachi)

1988 Stock Trading Expert System (Yamaichi Security)

1989 LIFE (Lab. For Intl. Fuzzy Eng.) Japanese provides US70m on Fuzzy Research

1989 First Fuzzy Logic air-conditioner

1990 First Fuzzy Logic washing machine

1990 Japanese companies develop fuzzy logic application in a big way

2000?

# Current Applications of Fuzzy Logic

- Camera aiming for telecast of sporting events
- Expert system for assessment of stock exchange activities
- Efficient and stable control of car-engines
- Cruise control for automobiles
- Medicine technology: cancer diagnosis
- Recognition of hand-written symbols with pocket computers
- Automatic motor-control for vacuum cleaners
- Back light control for camcorders
- Single button control for washing machines
- Flight aids for helicopters
- Controlling of subway systems in order to improve driving comfort, precision halting and power economy
- Improved fuel-consumption for automobiles

# Current Applications of Fuzzy Logic

- Expert systems also utilised fuzzy logic since the domain is often inherently fuzzy.
- Some examples:
  - decision support systems
  - financial planners
  - diagnosing systems for determining soybean pathology
  - a meteorological expert system in China for determining areas in which to establish rubber tree orchards

# Current Applications of Fuzzy Logic

THE SENDAI SUBWAY SYSTEM

- First Proposed in 1978
- Granted Permission to operate in 1986 after 300,000 simulations and 3,000 empty runs

- Improved stop position by 3X
- Reduced power setting by 2X
- Total power use reduced by 10%
- Hitachi granted contracts for Tokyo subway in 1991

# Current Applications of Fuzzy Logic

## AIR CONDITIONER

- Initial design in April 1988
- Simulation in Summer of 1998
- Production in October 1989

- Heating and Cooling times reduced by 5X
- Temperature Stability increased by 2X
- Total power savings of 24%
- Reduced number of sensor

# Overview of Fuzzy Logic

- Study mathematical representation of fuzzy terms such as old, tall, heavy etc.
- This term don't have truth representation. i.e. truth or false [0,1]
- But, have **extended truth values** to all real numbers in the range of values **0 to 1**.
- This real numbers are used to represent the **possibility** that a given statement is true or false. (Possibility Theory)
- Example:

  The possibility that a person 6ft tall is **really tall** is set to 0.9 i.e. (0.9) signify that it is **very likely** that the person is tall.

# Overview of Fuzzy Logic

- Zadeh (1965) extended the work and brought a collections of valuables concepts for working with fuzzy terms called **Fuzzy Logic.**

- <u>Definition of Fuzzy Logic</u>

  A branch of logic that uses **degrees of membership** in sets rather that a strict true/false membership

# Overview of Fuzzy Logic

Linguistic Variables

- Fuzzy terms are called **linguistic variables.** (or **fuzzy variables**)

Definition of Linguistic Variable

- Term used in our natural language to describe some concept that usually has vague or fuzzy values

# Overview of Fuzzy Logic

- Example of Linguistic Variables With Typical Values

| **Linguistic Variable** | **Typical Values** |
|---|---|
| Temperature | hot, cold |
| Height | short, medium, tall |
| Weight | light, heavy |
| Speed | slow, creeping, fast |

# Overview of Fuzzy Logic

- Possible numerical values of linguistic variables is called **UNIVERSE OF DISCOURSE.**

Example:

- The **Universe of Discourse** for the linguistic variable **speed** in R1 is in the range [0,100mph].

- Thus, the phrase "*speed is slow*" occupies a section of the variable's Universe of Discourse. - It is a **fuzzy set**. (slow)

# Overview of Fuzzy Logic

<u>Fuzzy Sets</u>

- Traditional **set theory** views world as black and white.
- Example like set of young people i.e. children.
- A person is either a member or non-member. Member is given value 1 and non-member 0; called **Crisp set**.
- Whereas, Fuzzy Logic interpret young people reasonably using **fuzzy set**.

- **HOW?**

  By assigning **membership values between 0 and 1.**

# Overview of Fuzzy Logic

Example:

Consider young people (age <= 10).

If person age is 5 assign membership value 0.9

if 13, a value of 0.1

Age = linguistic variable

young = one of it fuzzy sets

Other fuzzy sets: old and middle age.

# Overview of Fuzzy Logic

Definition: Fuzzy Sets

- Let X be the universe of discourse, with elements of X denoted as x. A fuzzy set A is characterised by a membership $\mu_A(x)$ that associates each element x with degree of membership value in A.
- Probability theory relies on assigning probabilities to given event, whereas Fuzzy logic relies on assigning values to given event x using **membership function:**

$$\mu_A(x): X \rightarrow [0,1]$$

- This value represent the degree (possibility) to which element x belongs to fuzzy set A.

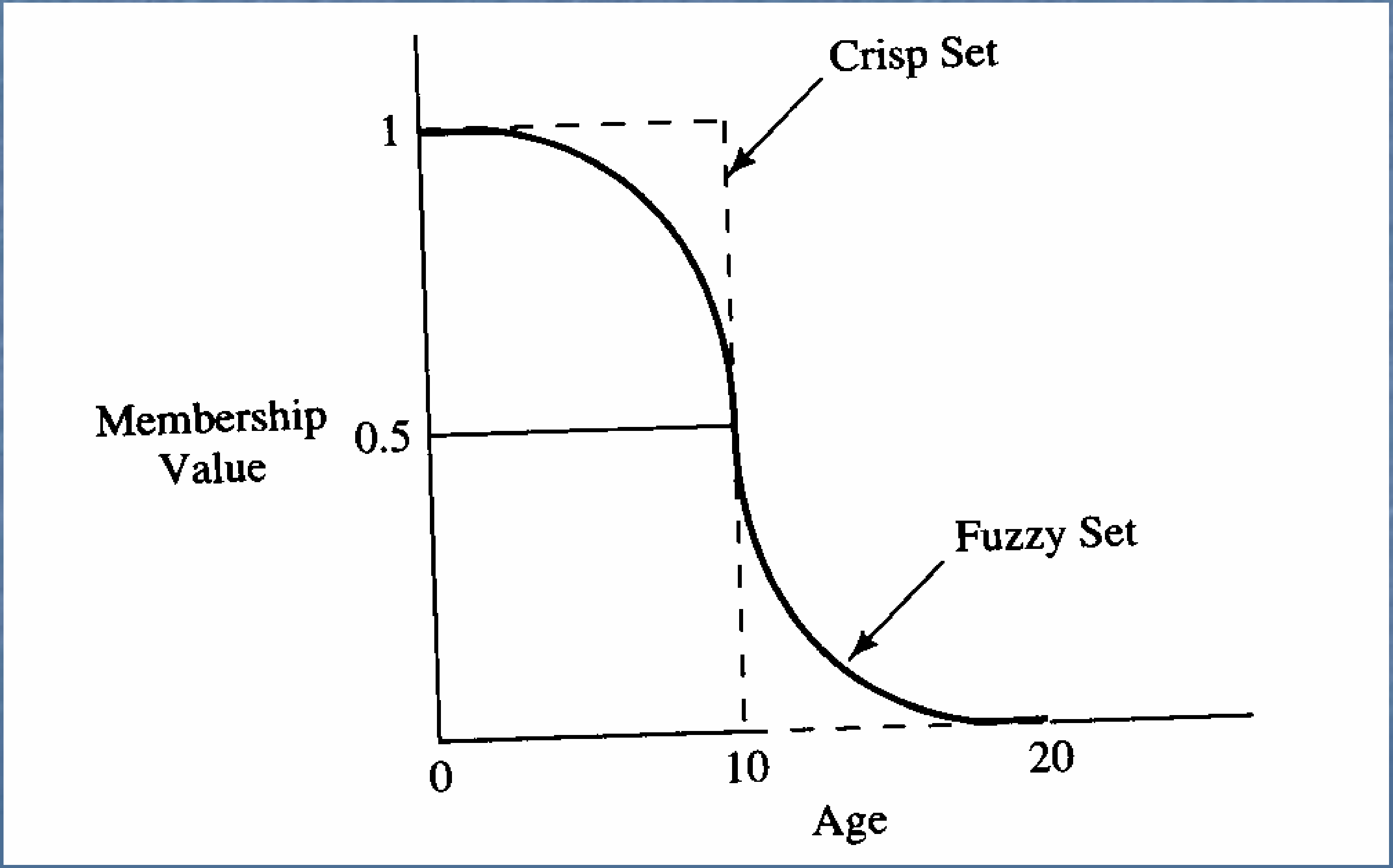
$$\mu_A(x) = \text{Degree}\ (x \in A)$$

- Membership values is bounded by:

$$0 \leq \mu_A(x) \leq 1$$

# Overview of Fuzzy Logic

Fig.1: Fuzzy and crisp sets of young people



# Overview of Fuzzy Logic

Fuzzy versus crisp sets

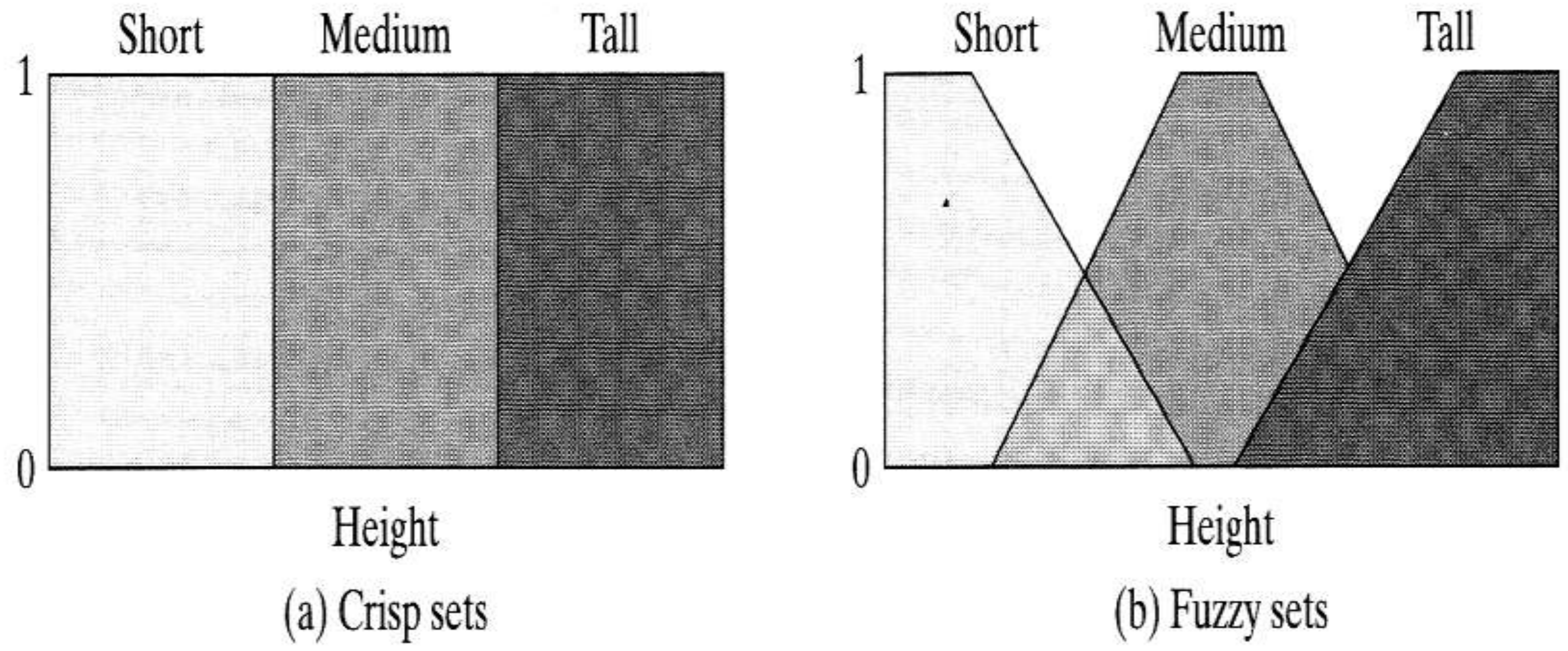



FIGURE 2.3: Fuzzy vs. traditional set membership.

# Overview of Fuzzy Logic

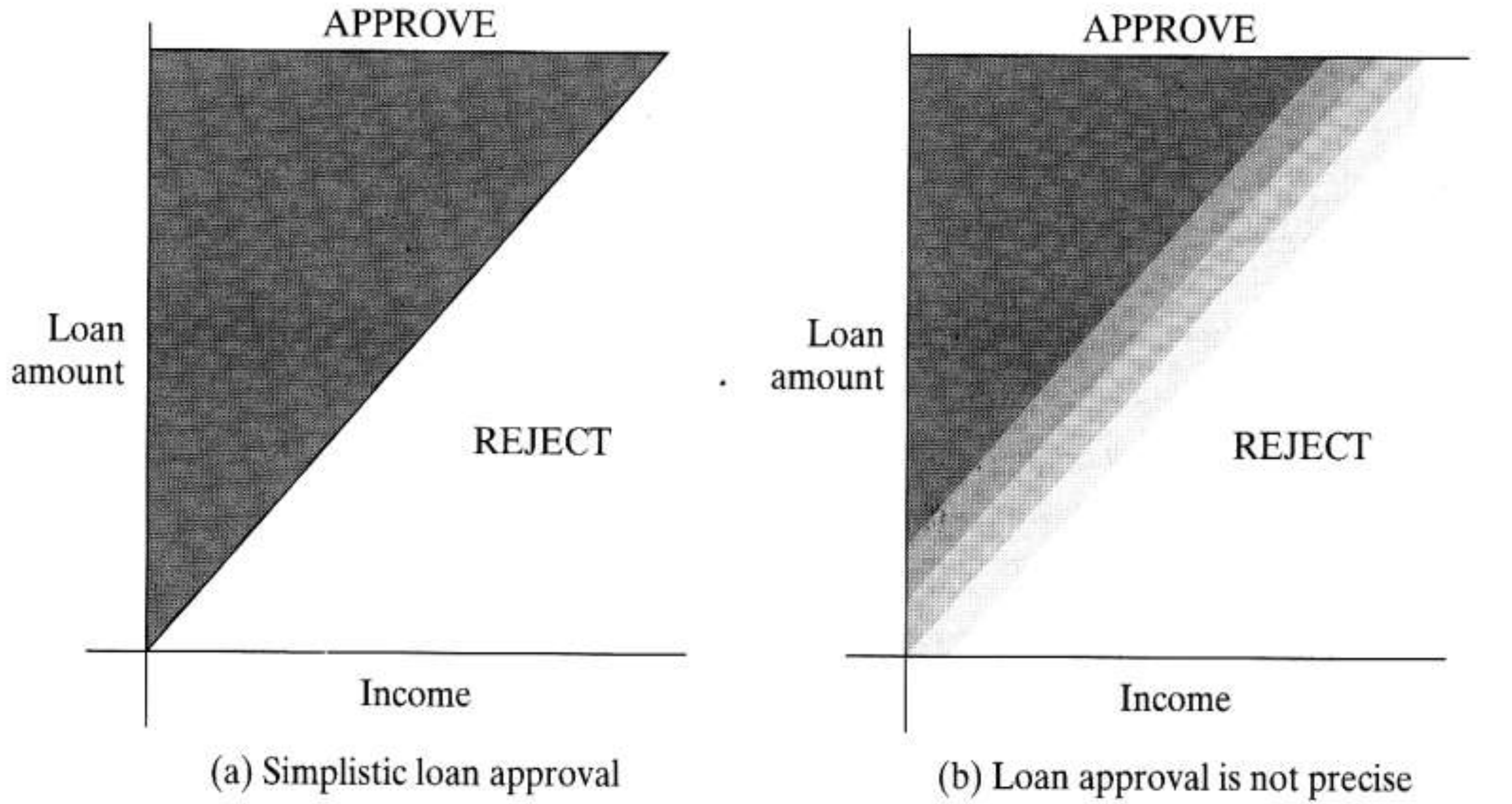



FIGURE 2.4: Fuzzy classification.

# Overview of Fuzzy Logic

Fuzzy rules represented in expert systems:

R1: IF Speed is **slow**
THEN make the acceleration **high**

R2: IF Temperature is **low**
AND pressure is **medium**
THEN Make the speed **very slow**

IF the water is very **hot** *(temperature)*
THEN add **plenty** of cold water *(amount)*
Fact: The water is moderately **hot**
Conclusion: Add a **little** cold water

# Overview of Fuzzy Logic

The difference between classical and fuzzy rules:

- IF-THEN using binary logic:

  R1: IF speed is > 100
  THEN stopping distance is long
  R2: IF speed is < 40
  THEN stopping distance is short

- IF-THEN using fuzzy logic

  R1: IF speed is fast
  THEN stopping distance is long
  R2: IF speed is slow
  THEN stopping distance is short

# Forming Fuzzy Set

- How to represent fuzzy set in computer??
- Need to define its **membership function.**
- One approach is:

  Make a poll to a group of people is ask them of the fuzzy term that we want to represent.

  For example: The term **tall person.**
- What height of a given person is consider tall?
- Need to average out the results and use this function to associate membership value to a given individual height.
- Can use the same method for other height description such as short or medium.

# Forming Fuzzy Set

## Fig. 2: Fuzzy Sets on Height

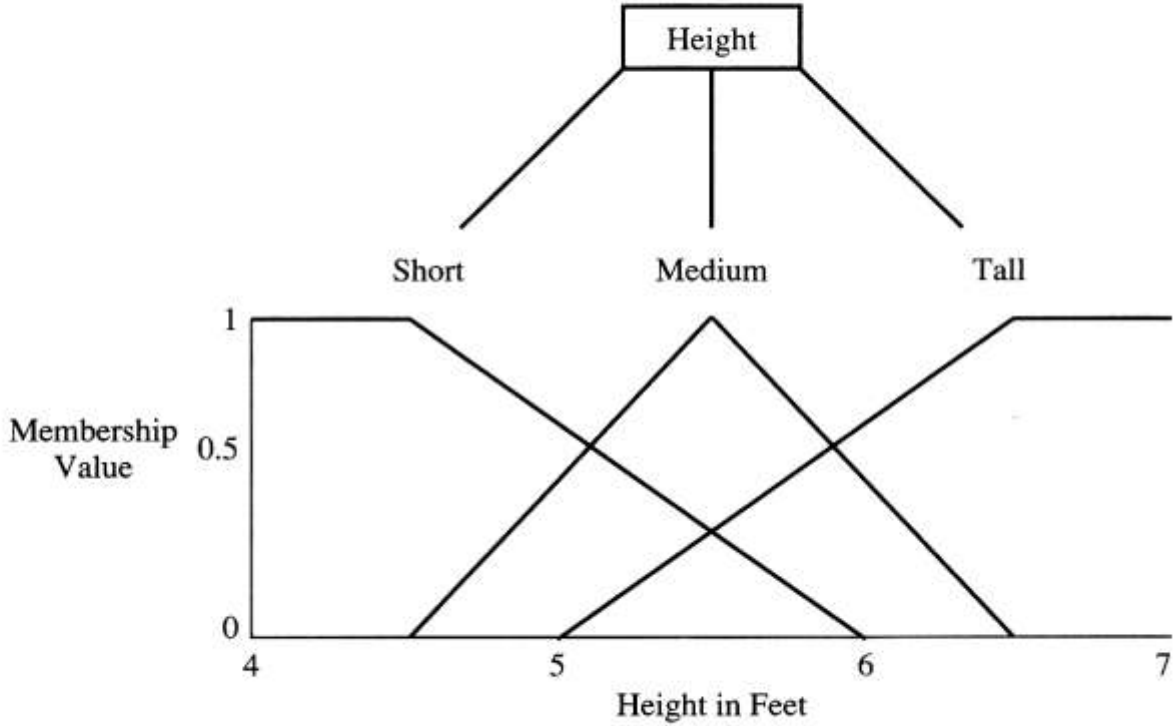



**FIGURE 13.2** **Fuzzy sets on height.**

# Forming Fuzzy Set

- Multiple fuzzy sets on the same universe of discourse are refers to as **Fuzzy Subsets.**
- Thus, membership value of a given object will be assigned to each set. (refer to fig. 2)
- Individual with height 5.5 is a *medium* person with membership value 1.
- At the same time member of *short* and *tall* person with membership value 0.25.
- Single object is a partial member of multiple sets.

# Exercise

a. Define some typical fuzzy variables
b. Define typical fuzzy sets for the fuzzy variables:
   i. Temperature
   ii. Weight
   iii. Speed
c. Define the universe of discourse for fuzzy sets in (b).
d. Draw each fuzzy set defined in problem (b).

# Fuzzy Set Representation

- How do we represent fuzzy set formally?
- Assume we have **universe of discourse** X and a **fuzzy set A** defined on it.

  **X = {x1,x2,x3,x4,x5...xn}**
- Fuzzy set A defines the membership function $\mu_A(x)$ that maps elements xi of X to degree of membership in [0,1].

  **A = {a1,a2,a3...an}**

  where

  **ai = $\mu_A(xi)$**

- For clearer representation, includes symbol “/” which associates membership value ai with xi:

  A = {a1/x1,a2/x2....an/xn}

# Fuzzy Set Representation

- Example: (refer fig. 2)

  Consider Fuzzy set of tall, medium and short people:

  TALL = {0/5, 0.25/5.5, 0.7/6, 1/6.5, 1/7}

  MEDIUM = {0/4.5, 0.5/5, 1/5.5, 0.5/6, 0/6.5}

  SHORT = { }

# Hedges

- We have learn how to capture and representing vague linguistic term using fuzzy set.
- In normal conversation, we add additional vagueness by using **adverbs** such as:

  *very, slightly, or somewhat..*
- What is **adverb**??

  A word that **modifies a verb**, an adjective, another adverb, or whole sentence.
- Example: Adverb modifying an adjective.

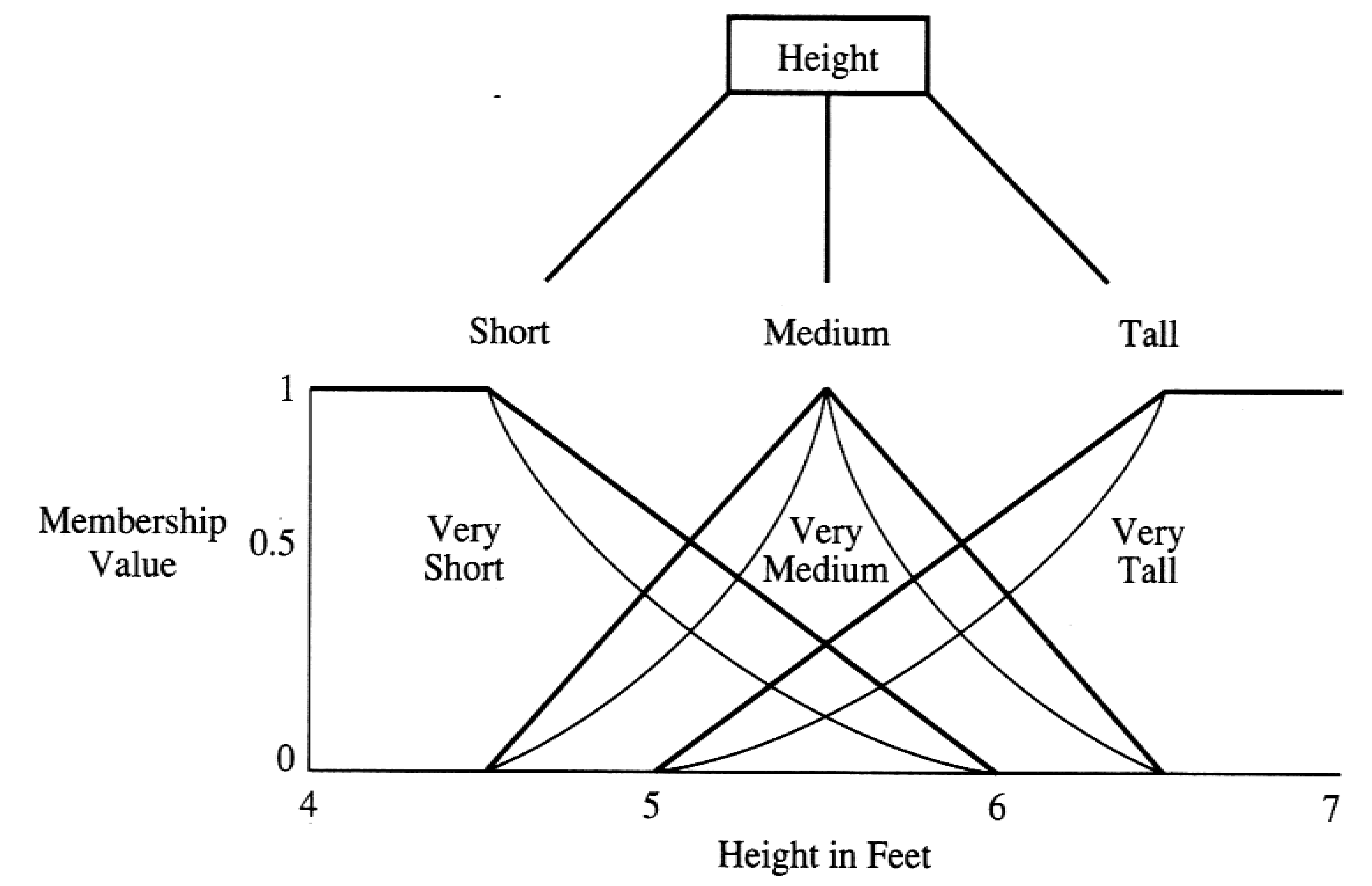
  The person is ***very*** tall

# Hedges

- How do we represent this new fuzzy set??
- Use a technique called **HEDGES**.
- A **hedge** modifies mathematically an existing fuzzy set to account for some added adverb.

# Hedges

Fig. 3: Fuzzy sets on height with ‘very’ hedge



# Hedges Commonly Used in Practice

**Concentration (Very)**

- Further reducing the membership values of those element that have smaller membership values.

$$\mu_{CON(A)}(x) = (\mu A(x))^2$$

- Given fuzzy set of tall persons, can create a new set of very tall person.
- Example:

Tall = {0/5, 0.25/5.5, 0.76/6, 1/6.5, 1/7)

Very tall = { /5, /5.5, /6, /6.5, /7}

# Hedges Commonly Used in Practice

**Dilation (somewhat)**

- Dilates the fuzzy elements by increasing the membership values with small membership values more than elements with high membership values.

$$\mu_{DIL(A)}(x) = (\mu_A(x))^{0.5}$$

- Example:

Tall = {0/5, 0.25/5.5, 0.76/6, 1/6.5, 1/7}

somewhat tall = { /5, /5.5, /6, /6.5, /7}

# Hedges Commonly Used in Practice

**Intensification (Indeed)**

- Intensifying the meaning of phrase by increasing the membership values above 0.5 and decreasing those below 0.5.

$$\mu_{INT(A)}(x) = 2(\mu_A(x))^2 \quad \text{for } 0 \leq \mu_A(x) \leq 0.5$$
$$= 1 - 2(1 - \mu_A(x))^2 \quad \text{for } 0.5 < \mu_A(x) \leq 1$$

- Example:

short = {1/5, 0.8/5.5, 0.5/6, 0.2/6.5, 0/7}

indeed short = { /5, /5.5, /6, /6.5, /7}

# Hedges Commonly Used in Practice

**Power (Very Very)**

- Extension of the concentration operation.

  $\mu_{POW(A)}(x) = (\mu_A(x))^n$

- Example:

  Create fuzzy set of very very tall person with n=3

  Tall = {0/5, 0.25/5.5, 0.76/6, 1/6.5, 1/7}

  Very very tall = { /5, /5.5, /6, /6.5, /7}

# Fuzzy Set Operations

Intersection

- In classical set theory, intersection of 2 sets contains elements common to both.
- In fuzzy sets, an element may be partially in both sets.

$$\mu_{A \wedge B}(X) = \min(\mu_A(x), \mu_B(x)) \quad \forall x \in X$$

- Example:

Tall = {0/5, 0.2/5.5, 0.5/6, 0.8/6.5, 1/7}

Short = {1/5, 0.8/5.5, 0.5/6, 0.2/6.5, 0/7}

$\mu_{tall \wedge short}$ =

Tall and short can mean medium

Highest at the middle and lowest at both end.

# Fuzzy Set Operations

<u>Union</u>

- Union of 2 sets is comprised of those elements that belong to one or both sets.

  $$\mu_{A \vee B}(X) = \max(\mu_A(x), \mu_B(x)) \quad \forall\, x \in X$$

- Example:

  Tall = {0/5, 0.2/5.5, 0.5/6, 0.8/6.5, 1/7}

  Short = {1/5, 0.8/5.5, 0.5/6, 0.2/6.5, 0/7}

  $\mu_{tall \vee short} =$

- Attains its highest vales at the limits and lowest at the middle.
- Tall or short can mean <u>not medium</u>

# Fuzzy Set Operations

Complementation (Not)

- Find complement ~A by using the following operation:

  $$\mu_{\sim A}(x) = 1 - \mu_A(x)$$

- Example:

  Short = {1/5, 0.8/5.5, 0.5/6, 0.2/6.5, 0/7}

  Not short = { /5, /5.5, /6, /6.5, /7}

# Fuzzy Inference

- **Fuzzy proposition:** a statement that assert a value for some linguistic variable such as ‘height is tall’.
- **Fuzzy Rule**: Rule that refers to 1 or more fuzzy variable in its conditions and single fuzzy variable in its conclusion.
- General form: IF X is A **THEN** Y is B
- Specific form: IF <u>height is tall</u> **THEN** <u>weight is heavy</u>
- Association of 2 fuzzy sets are store in matrix M called **Fuzzy Associative Memory (FAM)**

# Fuzzy Inference

- Rules are applied to fuzzy variables by a process called propagation. (inference process).
- When applied, it looks for degree of membership in the condition part and calculate degree of membership in the conclusion part.
- Calculation depends upon connectives: AND, OR or NOT.

# Fuzzy Inference

A fuzzy Logic program can be viewed as a 3 stage process:

**1. Fuzzification**

The crisp values input are assigned to the appropriate input fuzzy variables and converted to the degree of membership.

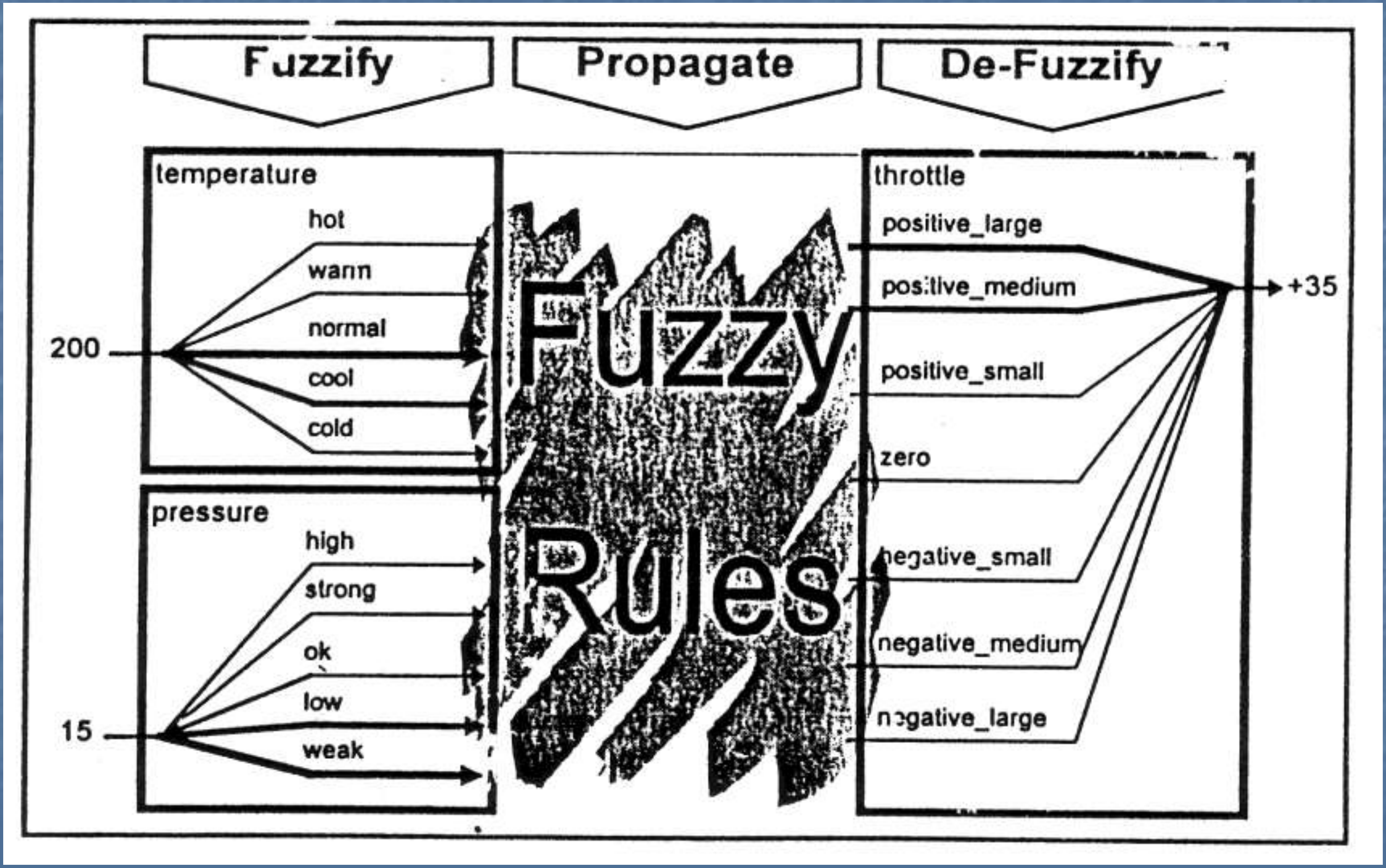
**2. Propagation (Inference)**

Fuzzy rules are applied to the fuzzy variables where degrees of membership computed in the condition part are propagated to the fuzzy variables in the conclusion part. (max-min and max-product inference)

**3. De-fuzzification**

The resultant degrees of membership for the fuzzy variables are converted back into crisp values.

# Fuzzy Inference



# Fuzzy Inference (Example)

Assume 2 cars travelling the same speed along a straight road. The distance between the cars becomes one of the factors for the second driver to brake his car to avoid collision. The following rule might be used by the second driver:

IF the distance between cars is very small
AND the speed of car is high
THEN brake very hard for speed reduction.

IF distance between cars is slightly long
AND the speed of car is not too low
THEN brake moderately hard to reduce speed

Identify:

Linguistic variables:

Fuzzy subsets:

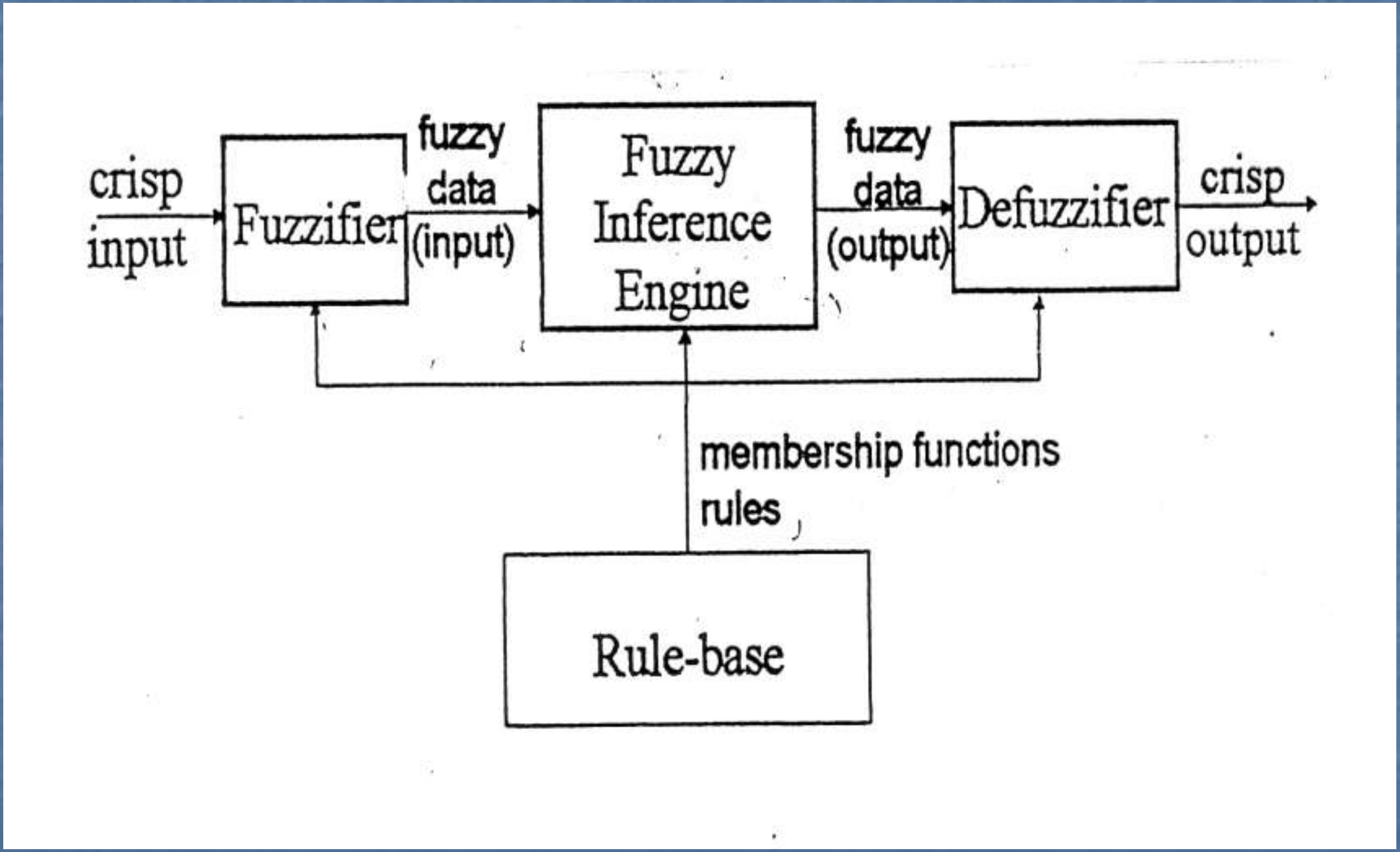
Connectives:

Hedges:

# Fuzzy Logic Controllers (FLC)

- Fuzzy Logic Controllers are build up from 4 main components:
    a. Fuzzifier
    b. Fuzzy inference engine
    c. Defuzzifier
    d. Rule base

# Fuzzy Logic Controllers (FLC)



# Exercise